The Impact of Context and Authenticity in Numerical and Mathematical Word Problems

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Contextualised or situated word problems, and the solving of the numerical and mathematical challenges that are contained in them, have evolved to become a staple and core component of the cultures of numeracy and mathematics instruction. Their universal popularity, longevity and pervasiveness, within the cultures of mathematics teaching and learning, can be explained by, and attributed to, their high-flexibility, multi-functionality and legacy as an effective format and conduit for teaching new mathematical content and for re-visiting and consolidating prior learning in this domain. Fore-courting and incorporating situational, contextual and authentic features and dimensions into the teaching and learning of numeracy and mathematics is highly recommended and espoused by various national and international organisations and in many mathematical studies and reports. The National Mathematics Advisory Panel (U.S., 2008) advocates lesson-modelling and instruction which features real-world contexts and authentic problem-solving. The California State Department of Education (1985) made the following recommendation to enhance problem-solving in mathematics and in numeracy: “problem contexts should be selected that closely resemble real-life situations”. The National Council of Teachers of Mathematics (U.S., 2000) highlights and stipulates, as an objective of mathematics teaching, contextualised and authentic learning opportunities that will develop understanding in students of the use and importance of mathematics and numeracy in everyday life and in the workplace. The “Programme for International Student Assessment” (PISA)(OECD, 2007) contained a mathematics literacy test which assessed students’ abilities to apply their mathematical content knowledge and skills to a broad range of real-world problems. The "Trends in International Mathematics and Science Study" (TIMSS) (National Centre for Educational Statistics, 2009) investigated the skills and cognitive competencies of reasoning as applied and utilised by students in solving mathematical problems which are set in real life contexts. The indigenous "Project Maths” (DES / NCCA, 2000) innovation also emphasises and prioritises the use of connections to real-life contexts as a means of enhancing and supporting the teaching and learning of mathematics in second level mathematics classrooms. Its overall aim is “to teach mathematics in a way which promotes real understanding, where students can appreciate the relevance of what they are learning and its application to everyday life” (“Report of the Project Maths Implementation Support Group - June 2010”, NCCA).

To script this investigative essay, I sourced and trawled through a wealth of educational and mathematical research literature. Each of these research case studies used a distinct analytical scalpel to investigate the word problem genre and, specifically, the authentic, contextual and situational stimuli and nuances of numerical and mathematical word problems.

Verschaffel et al. (2000) define "word problems” in numeracy and mathematics as “verbal (or written) descriptions of problem situations wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement”. Gerofsky (2009) states that word problems involve the "use of language and story to pose mathematical conundrums - but the purposes of these conundrums, and their purported relationship to real-life have not remained stable over time”. Gerofsky provides additional insights into word problems by stating that: "word problems
constitute a written and pedagogical genre within mathematics education”. Gerofsky also provides the following information on the dynamics and nature of word problems: “in pre-algebraic societies, mathematical word problems were the only way available to establish mathematical generality, through the heaping-up of numerous examples in story form”. Gerofsky concludes that a primary function of numerical and mathematical word problems is "expressing generality through exemplification". Palm (2006) describes word problems as "textual descriptions of situations assumed to be comprehensible to the reader, within which mathematical questions can be contextualised". Palm also states that word problems "provide, in convenient form, a possible link between the abstractions of pure mathematics and its applications to real-world phenomena". Palm’s latter interpretation portrays word problems as mathematical tasks "dressed up" in a real-world context and for their solution students must "undress" these tasks and solve them. The scenarios and contexts embedded in word problems may be of different genres and kinds - including vocational practice and everyday life. These categories are further described and parsed in the OECD’s "Programme of International Student Assessment" (OECD, 1999) as "tasks that someone in an out-of-school setting is likely to be called upon to address". Gerofsky (2004) states that word problems are "closely related to ancestral or antecedent genres including parables and recreational mathematical puzzles". Greer et al. (2009) describe word problems as "story problems". Sakshaug et al. (2002) define a word problem in numeracy and mathematics as "a task that requires the learner to reason through a situation that will be challenging, but not impossible. In the infrastructure of a word problem, there is a hurdle that the learner cannot immediately see how to get over or round". Fox and Surtees (2010) state, that in order to solve a mathematical or numerical word problem, the student must be challenged to "think differently", to "extend their thinking about a situation in a way that is new or different", to “think beyond the point from where he / she started" and "make decisions about which strategies to use". The dictionary defines "heuristics" as the art of problem-solving, involving the use of cognitive reasoning, prior knowledge and past experience in order to support the process of learning. In his seminal thesis on problem-solving, Pólya (1945), advocates providing the student with regular opportunities to engage with real and authentic problems. Pólya contends that "the independent solving of challenging problems will aid the learner far more than the aphorisms which follow, although as a start these can do him no harm". Pólya adds that: "A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his student in routine operations, he kills their interest, hampers their intellectual development, and misses his opportunity. But if he challenges the curiosity of his student by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking".

The first references in research literature to "authenticity" and "authentic learning” date back to the beginning of the 20th century; however, Newmann et al. (1992) were among the first researchers to use the term "authentic" formally in the context of teaching and learning. Berge et al. (2004) locate the genesis and evolution of the seminal conceptualisation of authenticity within the specific domain of cognitive research that focused attention on the "holistic task" and the “gestalt” dimensions of a problem. Brown et al. (1989) locates thegenesis of an
alternative and rival conceptualization of authenticity in the realm of cognitive research and applied psychology which focused on the investigation and formulation of "situated theses of learning". This alternative thesis infers and states that learning occurs best - or perhaps only - within context and situation. According to this "situated" thesis, learning and performance are so closely related to context that the actual schema of the underpinning domain knowledge mutate and are different for different contexts. This "situated" thesis suggests that task performance, demonstrated in one context, may not be indicative of competence in another context. Cronin (1993) concludes that "authenticity is relative". Anderson et al. (1996) suggests that there is a continuum of theories of "situatedness". This continuum and constellation of "situated" theories include theories that posit and declare possibilities of transfer of learning across various contexts to those that argue complete contextual isolation and dependence. Complete contextual dependence means that the task could only be authentic if it accurately mirrored the specific context and location of the outside-of-school task. In this sub-thesis, generalisation or contextual simulation would not be possible or feasible. Other divergent sub-theses of authenticity refer to the permitted or desirable gradient of "fidelity" or "functional fidelity" that can be built into the word problem. “Functional fidelity” is defined as the degree to which a word problem statement imitates or mirrors the information, detail and stimulus-response options that are present in the real world. Palm (2008) defined "authenticity" as: "the concordance between, on the one hand, a mathematical school task that includes a description of a real out-of-school situation and, on the other hand, the actual real-life situation”. Vincente et al. (2007) claims that embedding a word problem in an authentic and realistic context can successfully activate and facilitate "germane processing which allows students to comprehend the presented information in a deeper and personal way". Greer et al. (2009) describe authenticity as "mathematizing situations and issues that connect with students’ lived experience". Kenk and Kian (2010) describe authentic stimuli and variables in word problems as conduits which "connect mathematical concepts, skills and strategies to purposeful, relevant and meaningful contexts, therefore promoting a deeper level of understanding in the classroom". Schonfeld (1989) states that an authentic word problem presents the student with opportunities to engage with "a task in which the student is interested and engaged and for which he wishes to obtain a resolution". Niss (1992) describes "an authentic extra-mathematical situation as one which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues or problems that are genuine to that area and are recognised as such by people working in it". PISA does not preclude the inclusion of virtual contexts to ensure authenticity in a problem construct, but the authors "emphasise the use of real-world problems with authentic contexts describing out-of-school settings and problems that someone in such settings would be called upon to address" (OECD, 1999). Oblinger (2007) states that with authentic word problems the concepts being applied and learned are always part of a much larger "learning event" and are directly linked in the learner's mind with social and personal circumstances and realities - the setting, the activities, the people. Oblinger concludes that the more encouragement and opportunities that the learner has "to become invested in material on a personal level, the easier it will be to assimilate the unfamiliar". Lowrie (2000) states that contextualised word problems provide opportunities for students to apply personal knowledge and social experiences to the problem context. Berge et al. (2004) use the following
alternative terms for authentic learning, namely: "naturalistic learning" and "situated learning".

Verschaffel et al. (2000) contend that students need regular and consistent opportunities to engage with "really real" authentic word problems in numeracy and mathematics and not "pseudo-realistic" or frill-versions of the real world. Palm (2008) postulates that "the higher the representativeness of the simulation is, the larger will be the proportion of students that make proper use of their real-world knowledge, when working with the word problem, and that will not suspend the requirement that their solutions must make sense in relation to the out-of-school situation that is described in the task". Palm also claims that, when an authentic and real-life context is embedded in the word problem, these students have opportunities to practice and enhance their higher-level thinking about abstract mathematical concepts, to practice and apply problem-solving skills and to experience mathematics in an out-of-school and simulated scenario. Fox and Surtees (2010) consider that the desired gradient and mix of authenticity, lived-reality, context and situation can be introduced into word problems in numeracy and mathematics by teachers ensuring that the context of the word problem is always connected to the student’s “interests, environment and experiences”. Gerofsky (2009) identifies the potential contained in authentic, situated and contextualised word problems, including offering memorable imagery that can act as a touchstone for teachers and learners in building and discussing abstract concepts in numeracy and mathematics. Research undertaken into the processes and dynamics of motivation, by Ryan and Deci (2000), indicates that the level of students’ engagement in a task is enhanced when the students experience the task as being "authentic" and "meaningful". Pearse and Walton (2011) describe how the formulation of connections and linkages, across the conceptual landscape of numeracy and mathematics, and particularly when extended to connect with the real-world experiences of the students, can significantly support concept and knowledge development in students. Pearse and Walton consider that "building connections is at the heart of numeracy". Greer et al. (2009) claim that engagement with word problems, which depict and describe authentic and realistic scenarios from the students’ lived-experiences, can support the development and enhancement of reasoning and problem-solving competencies in students. By being familiar with the presenting challenge and task, the student quickly begins to realise that he / she already possesses a memorised knowledge base, skills-set and experiential infrastructure that can significantly inform his / her cognitive and meta-cognitive processing and subsequent actions and computation which are required to arrive at a solution to this word problem. Encountering familiar and realistic stimuli - within the textual detail of the word problem, or accompanying graphics, can be the ideal catalyst for arousing the student’s interest, motivation and curiosity in the specific problem statement. This activation of the affective and motivational domain can significantly support and sustain the student’s cognitive processing, and subsequent action and computation.

Selter (1994) claims that, when opportunities are provided in classrooms for students to engage with realistic and authentic learning, participating students begin to recognise and appreciate the potential and importance of numeracy and mathematics as critical tools for interpreting and understanding the realities and nuances of society in which they live. Selter
describes the multiple opportunities for learning that are provided in this approach as "emergent modelling". Palm (2009) claims that engagement with authentic and realistic word problem-solving "facilitates the learning of necessary skills for being able to use - and critically examine the use of - mathematics outside the mathematics classroom" and, also, facilitates the "development of an experience of school mathematics as useful and powerful for solving meaningful task situations in life outside the mathematics classroom". Haylock (1991) identifies the potential for enhancing numeracy and mathematics teaching and learning that is contained in contextualised word problems because they provide the students with "purposeful activity" in a "meaningful context". Berge et al. (2004) state that, for students to perceive the task within the word problem as realistic or authentic, the problem must be grounded in their experiences and interests. Berge et al. also suggests that, even if the problem is not based on the student’s current experiences, it may still be perceived as authentic if it somehow relates to the student’s future plans or expected career. De Block et al. (2003) claim that, as long as they are meaningful, familiar and appealing to the students, "realistic contexts" do not necessarily – or always - have to refer to aspects of the "real", social or physical world of the student. These researchers conclude that it is not the level or amount of realism, in the literal sense, that is crucial or critical, but rather the extent to which it succeeds in getting students involved in the problem and in engaging them in situational and meaningful thinking and interaction. Cumming and Maxwell (1999) contend that, if the task is not of the kind that students would expect to undertake in life outside of school, either now or later, it is highly likely to be seen by students as being "contrived" and "unappealing".

Bruner (1960) states that students’ understanding of the underlying structure of the numeracy or mathematics problem can be developed and enhanced by presenting students with "meaningful" and contextualised problems. Atkinson and Raynor (1974) state that "motivation to achieve" is a function of the individual student’s desire for success, the expectancy of success and the quality of incentives provided. Well-designed and well-planned word problems in numeracy and mathematics can provide multiple opportunities for students to embrace success in learning and problem-solving. Norman and Schmidt (1992) suggest that matching and connecting students’ prior knowledge and interests to the nuances of the word problem is necessary and essential to stimulate and motivate students to spend sufficient time on their problem-solving task. Wenger (1998) argues that learning increases and is enhanced when the student develops a "thirst for learning of the kind that engages one's identity on a meaningful trajectory and affords some ownership of meaning". Wenger suggests that, in order to establish such ownership, teachers should plan and design word problems that relate to real-life situations. Kaput (1994) correctly states that it is not the word problem in isolation that enhances teaching - rather it is the students’ development of meaning which is activated through engagement with the word problem. Cleary and Chen (2009) state that students’ motivation to engage with the specific word problem task is enhanced when they perceive the task to be "interesting or valuable". Greer et al. (2009) contend that the student who is learning numeracy and mathematics (or anything else) comes with an innate drive for sense-making that should not be violated and these researchers believe that attention should be given in teaching mathematics to making connections with the students’ interests and lived experiences. Greer et al. concludes that, as teachers, we need
to break away from traditional classroom cultures that state "this is how it has always been done". In his investigation of students’ engagement with contextualised word problems, Barwell (2011), states that students predominantly focus on three key aspects or features of the contextualised word problem, namely: "scenario", "personal experience" and "mathematical structure" and states that these three aspects of students’ attention are interrelated. Barwell also provides the following insights into this matrix of interrelationships: "personal experience is used to make the underlying mathematical structure meaningful and to interpret the scenario of the word problem" and "understanding the structure of word problems is necessary to successfully mathematize the scenario".

Greer et al. (2009) correctly state that what teachers think and do essentially govern whether and how students will encounter "real-world connections" ("rwc") for the mathematics and numeracy that they learn in school. Constructivist educators and researchers encourage and urge a fundamental change in the role of teacher, from deliverer of knowledge to facilitator of learning. By extension, these advocates of constructivism and socio-constructivism highly recommend and encourage the consistent provision of realistic and authentic learning opportunities for students because of the myriad of constructivist learning opportunities that this reality provides.

Bottge (1999) compared the impact on student performance of engagement with two types of numerical and mathematician word problems, namely word problems that were framed in realistic and authentic context and alternative versions which contained no contextual or situational stimuli. Results from this research support the contention that students’ performance improves when they are provided with opportunities to engage with contextualised and situated word problems.

Schoenfeld (1992) suggests that, in the traditional classroom, most of the word problems presented in numeracy and mathematics were low-challenge, repetitious and routine exercises, which were used to provide reinforcement and practice on a particular mathematical technique, which had been previously demonstrated and modelled to the students by the teacher. In these traditional classrooms, priority was not ascribed to the requirement or practice of having authentic, situational and contextual stimuli in the word problems in numeracy and mathematics. Making real, relevant and rational connections between the word problem scenarios and the lived experiences and interests of the students was not common practice in these classrooms. On occasions, lip-service was paid to authenticity, situation and context in the creation and scripting of word problems; however Cumming and Maxwell (1999) describe this practice as temporary and superficial "dressing up" and "camouflaging" of traditional forms of tasks to make them appear authentic and contextualised. Berge et al. (2004) declare that this artificial and superficial "camouflage" usually occurs within an underpinning epistemology and culture where curricular content is prioritised and supportive context and authenticity are conceived as being incidental and distracting. The underpinning epistemological and paradigmatic tenets, edicts and orientations, of these traditional classrooms, placed emphasis on the acquisition of numerical and mathematical knowledge; the process and "use and application" dimensions of numeracy and mathematics were, at that time, deemed to be of secondary importance to content, rules
and fact acquisition of. Deliyianni et al. (2009) state that the recommended pedagogical approaches to teaching numeracy and mathematics were prescribed and determined not only by the stipulations of the underpinning epistemology but also by the matrix - or "didactical contract" - of implicit and explicit rules and conventions that operated within the cultures of teaching at that time.

Other empirical research studies (Schnitzer, 1993)(Newmann et al., 1996)(Ferretti et al., 1996)(Mc Robbie et al., 2001) have focused on the impact of authentic and contextual stimuli and variables, within word problems, on student performance - and other student related outcomes - in the numeracy and mathematics domains. These researchers list the following as observable outcomes in participating students’ performance and behaviour:

- the student began to develop his / her own sense of authenticity by aligning the problem statement to his / her own personal experiences, interests and understandings
- levels of motivation and desire to learn increased
- an improvement in the students’ numerical and mathematical literacy
- development of strategic thinking skills - including competencies in higher order thinking - and enhancement in modes of analysis and reasoning
- development of "portable" skills and competencies which facilitate and enable transfer of learning to new outside-of-school situations and across the curriculum
- positive impact on the affective and motivation domain: e.g. development of resilience and perseverance in students’ engagement with word problems in numeracy and mathematics
- development of skills and competencies in knowing when and how to use mathematical knowledge for representing and solving problems in practical and realistic situations
- development of cognitive connections between reality and mathematics

**Investigating Why Students Ignore Contextual Stimuli and Cues in Numerical and Mathematical Word Problems**

Many researchers have investigated, and sought explanation for, the provision by some students of "non-sense", "unrealistic", "unreasonable" and "irrational" answers to numerical and mathematical word problems despite being provided with various contextual, authentic and situational stimuli and cues within the word problem statement. These contextual cues and stimuli are provided not only to make the word problem statement more authentic and interesting for the students, but also to stimulate and encourage a process of cross-checking and cross-referencing both their estimated and final calculated answers with their experiences and observations of engaging with similar scenarios and computations outside of school. When students fail to engage in a process of regular and consistent cross-checking and cross-referencing, they are not availing of – or activating - their critical reservoir of prior knowledge and experiential insights which can be used as a beacon in determining if their estimated and final calculated answers "make sense" relative to their real-life experiences. Determining that an answer makes sense, and is plausible, is one of the core sub-processes of
numerical and mathematical computation; using contextual and situational knowledge and experience, to cross-reference and cross-check the plausibility and validity of the calculated answer, should also be conceptualised and encouraged as an integral and critical component of the computational processes.

Some of the foremost researchers, who investigated the problematic phenomenon in numerical and mathematical work problems of "students abandoning and suspending their sense making capabilities" and "not seriously considering familiar aspects of reality" include: Greer et al. (2009), Palm (2008), Cooper (1994), Berge et al. (2004) and Schoenfeld (1991). Reasons investigated and identified by these researchers for this high level of disconnect and dissonance between the out-of-school experiential knowledge of the students and the "unrealistic" and "nonsensical" responses and solutions, which they provide when engaging with formal word problem-solving, include:

- Sometimes the "everyday context" can be a hindrance rather than a support to learning by distracting the student and resulting in the student performing less well in this genre of problem-solving compared to his / her performance in solving problems that are presented in a non-textual mode and without context. Because of similarities to real-life experiences, the student may treat the word problem statement too realistically or literally and introduce considerations to his / her cognitive processing that are not appropriate to the given problem statement.
- Some students have been conditioned - by textbooks, classroom cultures and teachers’ stated and inferred value-sets - to formulate and possess the following subliminal assumptions and generalisations: (a) The word problem statement is always "self-contained" which means that there is always an inevitable and distinct connectivity and “linear relationship” between the variables that are presented in the problem statement. All that is required of the student is to identify the specific computational operation that is needed to solve this word problem. Unfortunately, this means that, once the required computational operation is superficially identified by the student, additional analysis of all the other contextual detail is ignored by the student. (b) Some students do not try to analyse and understand the entire dynamics and detail of the problem statement but simply engage in a "reflex-like" scanning and recognition of the keywords in the problem statement before "quickly jumping to the actual calculating work". Because some students perform "superficial analysis of the numbers and keywords provided in the problem text" followed by "unreflective calculations", they rarely give consideration, or practice fidelity, to the real-life aspects and variables of the scenario that are described in the word problem. In addition, some students may not possess the required level of real-world knowledge and detail that is necessary to activate and implement a process of cross-referencing between the final calculated answer and prior outside-of-school experiences.
- The students’ current motivational threshold and readiness phase may result in unwillingness in some students to invest the mental effort, dedication and perseverance that is required to engage with the various variables and stimuli that are embedded within the word problem.
Some students’ approaches to solving word problems can be significantly determined and scripted by "routine expertise" rather than by the required and more comprehensive "adaptive expertise". Hatano (2003) defines "routine expertise" "completing school mathematics exercises quickly and accurately without much understanding". Hatano also describes and documents the more holistic and comprehensive "adaptive expertise" as "the ability to apply meaningfully learned procedures flexibly and creatively". When students consistently ignore contextual stimuli and cues within word problems, they are opting to operate in "routine" mode and need to be encouraged to adjust to, and employ, "adaptive" mode.

Some challenges that are embedded in word problems are "stereotypical" and mere "stylised representations of hypothetical experiences" which do not require the students to engage in any in-depth analysis, reasoning and reflection. In this reality and scenario, some students have a high tendency to ignore the contextual and situational stimuli in the word problem.

Because the student does not possess adequate reading and comprehension skills, he/she as to focus predominantly on decoding the text and not on making sense of the problem statement.

Baruk (1985) investigated students’ engagement with the following word problem that contained contextualised stimuli:

"On a boat there are 20 sheep and 6 goats. How old is the captain?"

Baruk’s research involved the participation of students in senior classes in primary schools in France. His research findings indicated that many of these participating students made the subliminal and unreflective assumptions that connective linearity existed between the given variables of "20 sheep", "6 goats" and "captain’s age" and, also, that the situation and scenario described in the text could be un-problematically solved by opting to use the most-likely mathematical operation, namely addition. Once the required operation was identified, no further or retrospective analysis of contextual detail was required. Consequently, a significant number of these primary students stated that the captain’s age was "26 years".

Reed (1999) also cites the following as an example of an erroneous assumption and hypothesis: "it takes 10 times as much time to run a distance that is 10 times as long". Reed suggests that cross-referencing this assumption and statement with the student’s prior knowledge and experiences from real-life may raise the student’s awareness of the inevitability and reality that the runner will slow down over a longer distance because of the onset of fatigue and dissipation of stamina.

Greer et al. (2009) also investigated the trend and prevalence of "nonsense" responses in students’ responses to numerical and mathematical word problems. These researchers compared students’ (10 – 11 years old and 13 – 14 years old) attainment in "standard" contextualised word problems with performance in "problematic" contextualised word problems. These researchers defined "standard" contextualised word problems as problem statements in which the numerical or mathematical operation was reasonable and easily
detectable because of the contextual cues that were provided. They also defined "problematic" contextualised word problems as involving problem statements that required consideration of more subtle aspects of the situation described. The following "standard" contextualised word problem was one of the problems presented to participating students in this research:

"Pete organised a birthday party for his tenth birthday. He invited 8 boy friends and 4 girl friends. How many friends did Pete invite for his birthday party?"

The following is an example of a "problematic" contextualised word problem where analysis of the contextual detail would reveal some problematic assumptions and hidden complexities between the contextual variables:

"If there are 14 balloons for 4 children at a party, how should they be shared out?"

The research findings indicated that participating students performed significantly better when engaging with "standard" contextualised word problems. The level of "realistic responses" - where students analysed and utilised contextual and physical logistics - in framing their responses, was extremely low and fell consistently within the range 5 - 20%.

Following on their research into the "nonsensical" dimension of students’ responses in numerical and mathematical word problems, Verschaffel et al. (1999) contend that this tendency is prevalent in many classrooms.

Palm (2008) also investigated the rationale and explanation for students providing answers and solutions that are "inconsistent" with, and disconnected from, the real-world similarities end exemplars that are described in the word problems. Palm used interviews with participating students in order to acquire insights into this realm of inconsistent student behaviour. After completing his interviews, Palm concluded that there are two primary reasons why students provide nonsense answers to numerical and mathematical word problems, namely:

- There is a strong avoidance tendency in some students not to engage in the required in-depth analysis and linking with real-world scenarios to inform and validate their approaches to solving the word problem. The absence of this reflective, connective and analytical preparation - which could have activated a comprehensive decoding and investigation of the full range of presented stimuli, including contextual cues - means that some students were likely to select a computational strategy and approach which was incorrect and inappropriate and which would ultimately result in an incorrect solution to the word problem. Equally, in this scenario, no follow-up evaluation, validation and sense-checking of the computed answer is likely to be performed by the student.
- The other main reason for the student’s provision of a non-sense answer was his / her beliefs and values set about school mathematics and the role of word problem-solving within the numeracy and mathematics domains. These beliefs and values and set of "implicit rules of didactical contracts", that operated in traditional classrooms, where
it was inferred that when a student was engaged in solving word problems in numeracy and mathematics, he / she should exclusively and predominantly consider only what is written in the text and nothing else. This values set also inferred that any comprehensive analysis or consideration of real world applications or connections with the word problem statement, are not necessary to successfully solve a word problem in numeracy and mathematics.

Developing a Taxonomy and Index of Authenticity

Palm (2008) developed the following index or "framework of authenticity" to enable teachers to plan and incorporate the desired gradient and level of authenticity and "real-life fidelity" in the word problem text. Each of these documented variables and stimuli can be conduits and scaffolds for introducing the required gradient of authenticity and contextual reference into the word problem. Equally, this "framework of authenticity" can be used as an investigated lens to determine the precise levels of authenticity and contextual cues that exist within an externally-produced word problem. Palm listed the following variables and nuances as key indicators of authenticity:

(a) “Event” - for the event that is described in the word problem to be deemed authentic, it is a prerequisite that the event has taken place or has a fair chance of taking place.

(b) “Question” - the question posed in the word problem must be one that actually might be posed in a real-life event or situation.

(c) "Presentation" - refers to the way the textual message is conveyed to the students e.g. orally or in written form (using words, together with accompanying diagrams, graphics and/or tables). Since all students do not cope equally well with textual and written communication (Newman, 1977) or graphical representations or verbal interactions (Nathan and Kim, 2007), the preferred mode of representation can significantly and substantially influence student performance and attainment outcomes.

(d) "Language Use" - refers to the semantic, referential and stylistic aspects and conventions of numerical and mathematical texts.

(e) “Purpose of the Task” - the purpose of the task solving needs to be clear and make sense to the student.

(f) “Information and Data” - refers to the detail and information that is provided in the word problem and from which the solution to the problem can be identified.

(g) "Availability of Solution Strategies" - refers to the match and connection between the range of possible solution strategies that are available to the student and the suite of strategies that are available to the persons in out-of-school contexts use to solve corresponding or similar tasks.
(h) "Circumstances" - refers to the circumstances in the classroom or homework context under which the task is to be solved.

(i) "Availability of External Tools" - refers to the availability of suitable concrete and experiential tools that can support cognitive processes e.g. calculator, map, ruler, Internet etc.

(j) "Guidance" - refers to guidance from an external agency (e.g. teacher) in the form of explicit or implicit hints.

(k) "Collaboration and Consultation" - refers to the opportunities and possibilities of peer-assistance being provided to the student.

(l) "Discussion Opportunities" - refers to the possibilities and opportunities for students to ask about and discuss the meaning and challenges of the task.

(m) "Time" - refers to time pressure and time constraints under which the student has to operate. It is important that time constraints are such that they will not cause significant or stressful difficulties for the students.

(n) "Consequences of Task Solving Success / Failure" - refers to the impact of success or failure - in engagement with this word problem - on the intrinsic and extrinsic motivational thresholds of the student.

(o) "Solution and Validation Requirements" - refers to the student’s assessing and judging of the validity and accuracy of his / her final calculated answer to the word problem together with a follow-on discussion of the solution methodologies that were employed.

Palm used his taxonomy of authenticity to inform and script his subsequent research into the impact of authenticity on student performance in word problem-solving. In his research, Palm investigated and compared the target students’ responses when engaging, firstly, with a "control" version of the word problem, which contains a "low or less" variant and rating of authenticity and, secondly, with a version of the word problem which contains a "higher" rating of authenticity, which included additional situational and contextual detail. The following examples illustrate Palm’s dual investigative focus:

(1) "Anton has bought 4 planks of 2.5 m each. How many planks of 1 m can he saw out of these planks?"

(2) "You were building a cabin and as walls you want to use planks that are 1 m long. You are at the moment short of thirteen 1-meter planks. A friend says that she has found 4 planks, each 2.5 metres long. You were wondering if that is enough to finish the walls. How many 1-meter planks can you saw out of the planks she found?"

Students, who have encountered and experienced a similar genre and classification of problem in their experiences in-school or outside-of-school, are more likely to be aware and understand that the entire length of each 2.5 metres plank cannot be used in these tasks, given the required measurements for the sawed lengths of planks. These students are also likely to
be aware that "wastage" and "off-cuts" are inevitable outcomes in this scenario when each plank is sawed to the specified dimensions. Students, who do not possess - or who fail to activate - this prior knowledge or experience, face and encounter this word problem without possessing a critical, operational or experiential knowledge base which can be used as a beacon to script their planning, analysis and choice of action. In this "low or less authentic" variant and exemplar of the word problems, the description of a real-life task situation is kept to a minimum. The participating students are given very little information about the circumstances of the task situation, including an absence of detail on the purpose of the problem task. All of the "more authentic" task descriptors include a more thorough description of the task context so that the task can make sense to the student. The purpose of solving the task is also provided in the text of the word problem. For the word question and statement to be relevant to the student, and its purpose to be immediately apparent, the text in the more authentic task variant invokes the idea of "enough" planks and also includes information on the number of planks that is needed to complete the task of "building a cabin". Lengthy passages of text have been avoided in Palm’s word problems in order to support and facilitate students with reading difficulties to understand and engage with the required tasks.

Following on an analysis of his research findings and results, Palm concluded that "increased task authenticity, even when it has to be accomplished solely by a modification of the task text, can increase students’ tendencies to use their real-world knowledge in the solutions to word problems". Palm recommended that the situational and contextual stimuli be included in the planning and development of word problems that are to be used in the teaching and learning of numeracy and mathematics in the classroom.

Dapaee et al. (2010) developed an alternative index of authenticity which identified and documented specific "entry" and "exit" approaches and strategies that teachers can use to activate and enhance the authentic, situational and contextual features and dimensions in word problems. The following strategies and approaches were documented and recommended in that research:

- Re-word the word problem. Get the student to re-read and restate the word problem in his / her own words.
- Seek to clarify and define the meaning of the situations, ideas, objects, persons and occupations that are mentioned in the word problem.
- Build on and activate the student’s out-of-school experiences and knowledge. Link the word problem to a personal experience of the student. Refer to a related event that has happened in the real world of the student’s experiences.
- Take explicitly into account, and refer to, the realities of the public context from which the word problem is derived. Identify the conditions and assumptions of the real-world exemplar.
- Interpret and validate the problem outcome and solution by linking to real-life situations. Place the mathematical solution back into a real life context. Seek real-life explanations for the obtained computation solution.
• Try and identify corresponding real-life situations and scenarios. Refer to corresponding real-life applications and the practical relevance for learning and living that results from the student acquiring insights into, and familiarity with, the processes of solving this particular problem genre or class.

Oblinger (2007) also developed an additional index of authenticity by distilling the essence of the "authentic learning experience" down to its core elements. These include:

• Real-world relevance. The word problem at should mirror and be framed by authentic activities which match the real-world tasks in practice as nearly and genuinely as possible.
• Ill-defined or open problem. Authentic activities are relatively open to multiple interpretations, requiring students to identify for themselves the specific tasks and subtasks that are needed to successfully complete the holistic task requirements.
• Multiple perspectives. Authentic activities provide opportunities for students to examine the problem statement from a variety of conceptual and practical perspectives.
• Collaboration. Success is not always achievable by an individual learner working alone. Authentic activities can be used to facilitate cooperative learning.
• Reflection and meta-cognition. Authentic activities provide opportunities for learners to make choices and engage in reflection on their learning.
• Multiple interpretations and outcomes. Rather than yielding a single correct answer which can be obtained by the application of singular rules and procedures, authentic activities can allow for diverse interpretations and competing actions and solutions.

Berge et al. (2004) developed the following checklist of strategic questions and foci that can be used by teachers to plan, incorporate and evaluate the desired gradient and density of authentic challenge and contextual situation into a word problem:

• How authentic is the word problem statement? To what extent is the problem statement in touch with the students’ experiences and daily life? Is the problem that is presented a "personalised" one for the students? How familiar are the students with the problem type?
• Do the students consider the word problem statement to be realistic, interesting, genuine and worth finding a solution to? Does the task demand excessive effort and perseverance? Is the problem sufficiently challenging to motivate the students? Can the problem be solved within the available time-frame? Are there any excessively-challenging or hidden aspects to the word problem?
• Is the word problem "open"? Some students consider authentic problems particularly challenging because they require flexible or multi-step approaches.

In their seminal study, Vicente et al. (2007) investigated the impact of "conceptual" and "situational" modification and adjustment, in the text of word problems on student performance in numeracy and mathematics. This research also identified a range of strategies and approaches that can be used by teachers to create the desired level of authenticity and
contextual reference within a word problem. In their research, Vicente et al. defined situational adjustment and enhancement as "personalising" the word problem so as to provide the students with "enriched" problem contexts which represent real-world situations and scenarios. These personalised numerical and mathematical word problems were defined by the degree of "ordo naturalis", realism and similarity that they had with the students’ lived out-of-school experiences and interests. Equally, as part of situational modification and adjustment, additional pieces of information - such as making explicit the rationale, motives, settings, and temporal structure of the problem – were also frequently provided within a word problem statement. “Conceptual adjustment” was defined as - and attempted by - making more explicit and transparent the relationships that exist between the singular variables in the word problem and the underpinning conceptual and semantic infrastructures. Care was also taken not to alter or dilute the underlying semantic and mathematical structures of the problem statement.

In all three versions of the word problems - "situationally modified", "conceptually modified” and "standard and unmodified" – care was taken to ensure that excessive reading and comprehension demands were not placed on the learner by the readability level, syntactic requirements and complexity of the text. Care was also taking in ensuring that an equivalent level of decoding and comprehension challenge was presented by all three versions of the word problem. No time restriction was placed on students’ engagement with the word problems. The following is an example of the "unmodified and standard" version of a contextualised word problem that was used in the research:

"Peter has 37 metres of cable. He bought A metres of cable more. He used B metres of cable and he ended up with 11 metres of cable. How many metres of cable did he buy / use?"

The following version of the same word problem contains conceptual modification and reformulation; I have identified these modifications in italics:

"Peter has 37 metres of cable. He bought A metres of cable more and joined them with those that he had. From the resulting total of metres of cable he used B metres of cable and he ended up with 11 metres of cable. How many metres of cable did he buy / use?"

This third version of the word problem illustrates how situational rewording has been used to make the problem statement more user-friendly by highlighting the intentional, causally and temporal structure of the situation that is described in the word problem; I have again identified these modifications in italics:

"Peter wants to renew his house’s wiring. Peter still possesses 37 metres of cable, from a previous renovation. As Peter realises that these metres will not be enough cable for the whole installation, he bought A metres of cable more. After buying those metres of cable he began the renovation. While making the renovation he has used B metres of cable, and when he finishes he realises that there remains 11 metres of cable. Peter wonders: how many metres of cable have I bought / used?"
Students’ performance in engaging with modified and unmodified text was analysed and performance outcomes were compared. On completion of the research, Vincente and his team of research colleagues, concluded that conceptual rewording and adjustment had a more significant and "facilitating effect" on students’ performance and attainment, in the domain of word problem-solving, than enhanced contextual and situational rewording and reformulation. These researchers also noted that the nature and format of situational rewording and contextual embellishment often necessitated the use of extended and longer text passages. For some participating students, having to wade through longer- and often more linguistically-complex text, significantly increased the "fatigue effect" and simultaneously reduced the likelihood of these students persevering with the task and arriving at a successful solution. However, Vincente et al. suggested that "situational adjustment and enrichments" can be effectively and successfully embedded in the word problem framework by the use of supportive graphics and visuals.

Some Critiques

Contextualised word problems have been critiqued at many levels in research literature. Post-modernistic researchers contest and critique the contention of transparency and value-neutrality in the language that is used within contextualised word problems. Bakhtin (1981) claims that all language is value-laden and grounded in the specific "chronotype or simulacra" configuration from which it evolves. In Bakhtin’s conceptualisation, any "neutral" representation or modelling of reality from secular society is impossible because every human expression operates within generic universes of time, space, storyline, intentions and meanings and these are inescapably linguistically framed and culturally mediated through dialogue and interpretation of language. Baudrillard (1988) agrees with Bakhtin’s thesis and declares "the impossibility of any representation of the real" and further adds: "there is no equivalent of the world, no double, no representation, no mirror". Mason and Pimm (1984) also contend that mathematical models are necessarily neither transparent nor obvious matchings with, and reflections of, societal phenomena and must, by their nature, stress and ignore particular features of reality and live scenarios. Verschaffel et al. (2000) declare that some word problems are superficial and "pseudo-realistic" and are constructed as a mere veneer of real life. These researchers claim that: "rather than functioning as realistic and authentic contexts, inviting or even forcing pupils to use their commonsense knowledge and experience about the real world in the different stages of the process of solving mathematical-application problems, school arithmetic word problems are perceived as artificial, puzzle-like tasks that are unrelated to the real world". Bernstein (1990) also identifies some of the difficulties in planning and constructing appropriate word problems that are un-biased, politically-correct and value-neutral. Bernstein articulates some of these challenges by highlighting how social class and gender script, differentiate and inform students’ constructions and interpretations of knowledge as presented in word problems. Palm (2009) advises that: "different aspects of real-world situations affect students’ behaviour differently and the way they affect them may also vary between situations". Palm also states that some contextual and situation aspects will be "more important for student behaviour than others
and the degree of importance will also vary between situations”. Lave (1988) states that developing connections between the students’ out-of-school and classroom numeracy and mathematics is problematic and difficult to plan because the students’ learning contexts and profiles are individualistic and differ significantly. Berge et al. (2004) state that the current and contemporary conceptualisations of authenticity and context might still be too generalised and not far enough developed to serve as a definitive guide in the planning of all numerical and mathematical word problems. Deliyianni et al. (2009) claim that some students have difficulty engaging with word problems because they place their focus on the syntactic and linguistic structures within the textual framework and ignore the real meaning of the facts and situations that are described in the problem statement. Kajamies et al. (2010) and Vauras et al. (1999) agree that word problem-solving in numeracy and mathematics poses particular and specific challenges and difficulties for the low achieving student who regularly exhibits poor self-regulation competency and motivational vulnerability.

Despite these substantial and on-going criticisms and critiques, there appears to be a growing consensus among mathematics teachers and educational planners that the primacy and hegemony of word problems - and word problem solving – that currently exist within some cultures and strands of numeracy and mathematics need to be consolidated and extended so that they emerge as the status quo and will remain thus well into the foreseeable future. Tienken and Maher (2008) articulate a compelling case for the consolidation and expansion of word problem-solving as a core dynamic and orientation within all the cultures and strands of mathematics teaching and learning by reminding us that a key and fundamental objective of all formal education is "to help develop students who can think critically and solve authentic problems". As teachers, we need to re-discover the immense potential that lies within contextualised and authentic word problems to support and enhance the teaching and learning of numeracy and mathematics, not only in the mathematics class but across the curriculum. Dapaepe et al. (2010) also acknowledge the key position and potential of word problems, within the numerical and mathematical domains, and suggest that word problems present teachers with multiple opportunities to activate and incorporate a range of situational and contextual stimuli and cues which can encourage and facilitate sense-making in students. However, Dapaepe and his co-researchers lament the fact that some teachers do not always seize these available opportunities. Fox and Surtees (2010) warn us against reverting to the previous and constrained conceptualisation of word problems which defined the primary role and function of a word problem as providing practice in routine, mechanistic and repetitious computational operations.

It is important to provide opportunities for students to engage with word problems that are drawn from their own lives and, also, from contextualised situations and scenarios which arise in other subjects across the curriculum. The challenge for us, as teachers, is to continually establish inclusive learning environments in our classrooms that encourage students to "personalise learning" in ways that allow individual learners to extend, adapt, revise and adopt the formal numerical and mathematical ideas to a context in which they can place themselves. Barwell (2011) states when students are requested to write their own word problems in numeracy and mathematics, they are quite capable of “mathematizing”
situations and scenarios, which are based on real-world insights and considerations. The on-going challenge for teachers is to draw on this reservoir of skills and competencies in students so that they can respond appropriately and successfully to word problems they have not seen before. As teachers, we want our students to become efficient and successful problem solvers in numeracy and mathematics; consistent and regular engagement with contextualised and authentic word problems can be a critical scaffold and support on the journey to creating this reality.
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